A model of carbon dioxide flow

- $CO_2$ dissociation curve: Carbon dioxide is transported in the blood from the tissue to the lungs in three ways
    - dissolved in solution
    - buffered with water as carbonic acid
    - bound to proteins (particularly haemoglobin)
- Partial pressure of $CO_2$ is 5.3 $kPa$ in arterial blood and 6.1 $kPa$ in mixed venous blood. Arterial blood will contain about 2.5 ml per 100 ml of dissolved $CO_2$ and venous blood 3 ml per 100 ml.
- Cardiac output: 5 $litre\ min^{-1}$ carries 150 ml of dissolved $CO_2$ to the lung, of which 25 ml will be exhaled.
- The carbon dioxide concentration in capillaries is given by

$$C_{CO2} = 462e^{(0.00415P_{CO2})} - 340e^{(-0.0445P_{CO2})} \tag{1}$$

Inspired air flowing at a rate of $V^{in}_{air}$ carries carbon dioxide with a known colume fraction of $F^{in}_{CO_2}$ (0.005) giving a $CO_2$ flux of $V^{in}_{air}F^{in}_{CO_2}$. Expired airflow $V^{Ex}_{air}$ is assumed to be the same as inspired airflow and carries a volume fraction of $\frac{P^l_{CO2}}{P_B}$ and hence $CO_2$ flux of $V^{Ex}_{air}\frac{P^l_{CO2}}{P_B}$ where $P^l_{CO2}$ is the partial pressure of carbon dioxide and $P^B$ is the altitude-dependent atmospheric pressure. The net total carbon dioxide flux expired by the lungs will be

$$V^l_{CO_2} = V^{Ex}_{air}\frac{P^l_{CO2}}{P_B} - V^{in}_{air}F^{in}_{CO_2} \tag{2}$$

or

$$Q^l_{CO2} = V^{Ex}_{air}\left(\frac{P^l_{CO2}}{P^B} - F^{in}_{CO2}\right)/f_{CO2} \tag{3}$$

Within the alveoli we assume the flux across the membrane to be

$$Q^l_{CO2} = D^l_{CO2}\left(P^{lc}_{CO2} - P^l_{CO2}\right) \tag{4}$$

where $D^l_{CO2}$ is the bult carbon dioxide diffusivity in lung tissue.

The $CO_2$ concentration in the lung capillaries $C^{lc}_{CO2}$ is given by

$$C^{lc}_{CO2} = 462e^{\left(0.00415P^{lc}_{CO2}\right)} - 340e^{\left(-0.0445P^{lc}_{CO2}\right)}. \tag{5}$$

If the heart is pumping at a rate of $V_b$ the carbon dioxide flux carried into lungs by venous blood flow from muscle capillary bed where $CO_2$concentration is $C^{mc}_{CO2}$ is $C^{mc}_{CO2}V_b$. The carbon dioxide flux carried by the blood from lungs is $C^{lc}_{CO2}V_b$. The difference will be the flux due to respiration.

$$Q^l_{CO2} = V_b\left(C^{mc}_{CO2} - C^{lc}_{CO2}\right) \tag{6}$$

The $CO_2$ binding relationship also holds for muscle capillaries

$$C^{mc}_{CO2} = 462e^{(0.00415 P^{mc}_{CO2})} - 340e^{(-0.0445 P^{mc}_{CO2})}. \tag{7}$$

The CO2 flux out of the muscles $Q^{m}_{CO2}$ under steady state conditions is the difference between arterial and venous fluxes and must therefore be the same as the CO2 flux leaving the blood stream to the lungs:

$$Q^{m}_{CO2} = V_b \left(C^{mc}_{CO2} - C^{lc}_{CO2}\right) = Q^{l}_{CO2} \tag{8}$$

The difference between the partial pressure of CO2 in the muscle capillary bed and the muscle drives the CO2 flux from the production site in the mitochondria.

$$Q^{m}_{CO2} = D^{m}_{CO2} \left(P^{m}_{CO2} - P^{mc}_{CO2}\right) \tag{9}$$

where $D^{m}_{CO2}$ is the bulk CO2 diffusivity in muscle tissue. Lastly We model the CO2 metabolism via

$$Q^{m}_{CO2} = 462e^{(0.00415 P^{m}_{CO2})} - 340e^{(-0.0445 P^{m}_{CO2})}. \tag{10}$$